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OUTLINES OF PHILOSOPHIC EDUCATION.

On the extension of this mode of Teaching.

IT will require but little reflection to satisfy a candid mind, that the method of teaching which is found successful, in the earlier branches of education, will, with a few modifications, apply to the more advanced departments of it; for it is abundantly manifest that, as the same faculties of the mind are employed at every stage of philosophic investigation, the same principles of reasoning must be called in, to guide their operation, and a similar mode of training be adopted to invigorate their powers. In passing from one department to another, there is only a change of object, a different set of materials to work upon; the intellectual instrument, the mental proceses of analysis, discrimuation and arrangment remaining unchanged. Whether the subject matter upon which the attention is fixed, be Logic, Ethics, or Physics, the method of teaching will imply only a slight change, as to the way in which the professor exerts his talents and influence on the one, and in which the students employ their abilities and industry on the other.

Perhaps, the greatest difficulty is already overcome, and the chief obstacle to this plan of instruction removed, by the exertion made in the elementary classes, to form habits of *thinking*, and, to create a love for study. In the formation of all habits, whether bodily or strictly mental, the first efforts are the least agreeable, and require, of course the most unremitting endeavours on the part of both *teacher* and pupil whence it follows, that, whatever progress is made at this stage, in acquiring the use of the intellectual powers, and in deriving pleasure from their exercise, will proportionably facilitate the advancement of the student in all his subsequent pursuits. His acquisitions in this field may be regarded as a free stock, ready to be employed in any future undertaking.

Still, there is some reason to apprehend, that the indolence or vanity of teachers will supply them with powerful objections to this plan of study. The daily examinations of the students, on the subjects discussed in their hearing, and the perusal of their *themes* in private, with the view of reporting on them in the class, are employments of more drudgery than fame, and afford not the same opportunity for the display of technical learning, eloquence, or ingenuity, as it is enjoyed by him, who is known only to his pupils in the character of a *divine*, *doctor*, *lawyer*, or a mere *lecturer*. To submit to the labour of teaching would be to put himself on a level with the industrious teacher and artist, who are compelled to sacrifice a large portion of their time to the fatiguing duty of using all kinds of precept and example, with the view of instructing the learner to perform what they themselves profess, and what they undertake to communicate. But, without repeating what has been again and again said on this head, we still maintain that, without imitating, to a certain extent, the examples now mentioned, no tutor or professor can discharge his duty with facility and success.

The professors of moral and natural philosophy enjoy considerable advantages, in carrying on, in their respective classes, a plan of study similar to that which has now been described. The subjects of their lectures present to the minds of youth, comparatively stronger motives and allurements to study. The various powers of action which are analysed in the ethic class, such as instincts, appetites, desires, affections, and passions; the origin of moral sentiments; the developement of those energetic principles which animate and impel the vast mechanism of human society; the grounds and distinctions of good and evil, of praise and of blame; the sources of depraved taste in the public mind, with the remedy, or antidote; the opinions of learned men in ancient and modern times, relative to the obligation of virtue; the qualities of mind and of action, in which it consists; and the ultimate standard of excellence, as applicable to all ages and nations—these, and other similar inquiries, take a firmer hold of the youthful imagination than abstract discussions on the generation of ideas, and on the means of intellectual culture. Theoretical views and speculations too, on the principles of government, of jurisprudence, and the history of civil society, afford ample scope for eloquence, and admit of various and very interesting illustration. In like manner, the subjects of natural philosophy, the beauty, the grandeur, and sublimity of the material world, are well calculated to gratify curiosity, and to prompt investigation. The application of analysis and induction to the phenomena of *nature*, and to the interpretation of her laws, cannot fail to prove a delightful employment to the inquisitive student: and, in one word, the professors or tutors will find no difficulty in arranging a proper system of exercise and mental culture, suited to the circumstances of the young persons under their charge.

The proficiency which the students are supposed already to have made in the acquirement of knowledge, and of intellectual habits, may perhaps, rea-

der daily examinations, in this case, less necessary, and, at the same time, suggest some alterations in the mode of prescribing exercises. The essays written in the *ethic* or *physical* classes ought for example, to be longer and more difficult than those to which the young persons were accustomed in the elementary classes, proceeding on a small stock of materials, supplied by the professor, and subjected to less indulgent criticism, both as to matter and style of composition. Still the teacher, even there must not trust too much to any appearance of superior advantage, as connected with the subjects of his course or with the previous acquirements of his pupils. He must not cease to use all the means in his power to stimulate their efforts and maintain their industry. He must remember that their intellectual habits are not yet fully confirmed; that they are naturally averse to labour; exposed to many temptations; and not always sufficiently masters of their determinations, to follow out the good resolves which they may have made. The former course of discipline, therefore, is not to be hastily abandoned. Its essential parts, and particularly the unceasing watchfulness which it exercises over the conduct of the student, and the distribution of his time, ought to be firmly retained, and merely accommodated to the gradual change of circumstances, in which maturer age, and extended knowledge are understood to place him. The spirit of emulation must not be allowed to expire; nor ought the feeling of responsibility, in relation to the professor and the class at large, to suffer any diminution.

We have much satisfaction in stating, that a very great number of the most respectable teachers, and the professors of moral, and of natural philosophy, approving of this system of teaching, have had full experience of its numerous advantages, and are in the constant practice of examining their students, and of prescribing a regular course of themes and exercises, on the subjects of their respective lectures.

It is hardly necessary to observe, that this plan of teaching is equally applicable to the study of every branch of science.—In truth, whatever may be the subject of inquiry or pursuit, there is only one way of teaching it, namely, that of communicating suitable knowledge of *facts* and *principles*, which constitute the subject matter of research, and of rendering the means of intellectual exertion and culture, to the student.

It is to be expected that a considerable number of students now under consideration, are far from being advanced; their education in many instances, has neither been regular nor complete; their minds have not been improved; nor have their habits been formed on systematic industry; on which account, they stand very much in want of the direction and vigilant superintendance, which constitute the discipline of the junior classes. But, granting that they were of an age considerably advanced, there appears no good reason why they should be deprived of the manifold advantages of this practical tuition. It is not meant, indeed, that the same number, or even the same kind of exercises should be prescribed in the higher classes, that are regularly enjoined by the tutors in the elementary processes. It is merely intended that such a mode and course of practical teaching should be adopted, as will best promote the purposes of study, secure attention to the lectures, and lead the student to acquire a minute and familiar knowledge of all the subjects brought before him. In whatever circumstances lectures are delivered for the instruction of youth, the system of education is eminently defective, if it is not followed up with a regular examination, for, even a class of philosophers would give more attention to a scientific discourse, did they know that they would be called upon to render an account of it, either in conversation or in writing, by any one vested with authority to demand such a proof of their application.

We have heard it mentioned, as an obstacle to the extension of this system of teaching, that the recitation of the student's lessons, and the tutor's lecture, occupy the whole time allotted to the business of the classes, so that no time remains for examinations and themes. This reason, however, can only be of weight in the absence of all means of adding another hour to the daily employment of teacher and pupil, to be devoted entirely to the exercises just mentioned: even, if a second hour be not sufficient, let a third be appropriated, and the ground of the objection will be removed. The apology now alluded to, so little creditable to the judgment and zeal of those who urge it, is indeed but a branch of that foolish prejudice which makes philosophic education consist wholly in the mere communication of knowledge, and without any reference to the more valuable acquisition of intellectual habits, which enable the student to acquire knowledge of his own. Nothing, it is presumed, can be more obvious than that, in these days, when scientific publications are so generally diffused, the chief object with a professor ought not to be the *out-pouring of his whole stock of knowledge* into the minds of his pupils, but rather the exhibition of a sketch, or *outline*, of what ought to be known in his particular department, to be filled up by the private studies of the students, and to be used by them as a guide in their future inquiries. The great point with a teacher is to habituate and inure his pupils to think for themselves; to investigate, analyse, compare, and reason consequentially; and these habits are not so likely to be acquired in any other way as by regular examinations, and frequent compositions, on the various subjects which are laid before them. It is of much consequence that the teacher take notice of what his pupils are doing, every evening and morning; that he ascertain the

extent of their progress, and, above all, their success in acquiring the use of those intellectual instruments by which all real knowledge must be sought for, and obtained.

We conclude this section with observing, what we believe has been already observed, that the neglect of examinations and exercises by the professor is a great loss to himself, as well as to his pupils. He must forever remain a stranger to the art of *teaching*, and content himself with the reputation of a *mere hearer of lessons;* without the faculty of communicating knowledge to his pupils. It is only by experience that he can learn how to accommodate himself to the opening capacity of youth, and how to reconcile their minds to the irksomeness of labour, in following out the more difficult tracks of science. He who has no deeper acquaintance with his pupils than that which he gains by hearing them repeat the words of a book by heart, or that by delivering to them a *lecture* every day, will not at the end even of a long life, be much better qualified to charge the proper duties of a teacher, than on the day that he entered upon his office.

It is a great misfortune to the youth of our country, that too many of the teachers are very little wiser than the children under their instruction.—Abilities and experience combined with exalted worth, are not sufficient to secure to professional and accomplished teachers an introduction into our schools. The illiterate itinerant, however depraved, is too often selected to take charge of a school, because he will teach for a *trifle* less, per month. Parents! this is doing injustice to your children;—select judicious instructers, and you will satisfy your own consciences. Let no father neglect this duty—God enjoins it upon him.

ON THE FURTHER EXTENSION OF THIS MODE OF TEACHING INTO OUR SCHOOLS GENERALLY.

As the teachers of youth, in all seminaries, profess to have in view the same great end, and are supposed to be at all times ready to direct their endeavours, so as the most effectually to enlarge the knowledge, and to improve the talents of such as are committed to their care, we feel encouraged to suggest, for the consideration of our fellow-labourers in every part of the United States, a few points on which it seems practicable to amend the system of education, and to accommodate it somewhat more to the spirit of the times in which we live.—We are the more readily induced to enter on this step, so apt on many occasions to be deemed invidious, that we have already repeatedly disclaimed all pretensions to the merit of discovery, whether as to the theoretical views, or even as to the method actually pursued for calling these views into action. We profess to have accomplished nothing more than to have adapted to the elementary branches of education, some parts of that practical system of teaching which is acted upon in the Pestalozzian schools; and, indeed, exemplified in every other department of human pursuit, where proficiency depends upon the formation of habits. In imitating, however, the simple but comprehensive Pestalozzi, the common craftsman and the artist, it strikes us, that we approach more nearly to the method dictated by nature; for, as in the case of corporeal habits, so in those that are more strictly mental, facility and excellence are to be acquired in no other way, than by regular and persevering efforts. Viewing the matter, too, on general grounds, when we reflect that the period at which the systems followed, in this country, were introduced, was not distinguished either by general knowledge, or by very just notions relative to the main object of education, and, as it is admitted, that, since these times, much valuable light has been thrown upon every other field of human exertion, it is reasonable to expect, that the modes of teaching may, in like manner, be within the reach of a gradual and progressive improvement.

To give full effect, however, to the system we have ventured to strike out in this journal, we beg leave to premise that the office of teacher should be professional and permanent. He should be possessed of learning and other qualifications, and his salary adequate to his exertions and usefulness.—Such an arrangement seems absolutely essential to proficiency and success, in the art of teaching; for this art, like all others, being founded *on practice and observation*, must derive, from that quarter, all the improvement of which it is susceptible. Upon the erroneous supposition, that the art of teaching consists in the mere communication of knowledge, it has been inferred, that wherever a person has acquired a certain portion of science and literature, he is immediately qualified to instruct others. But knowledge and intellect are not the only qualifications of a teacher, nor even the most important part. On the contrary, it is sufficiently confirmed by experience, that the most profound scientific attainments, the finest imagination, and the most exquisite taste, do not, of themselves, qualify their possessor for becoming a discriminating and useful teacher. The knowledge which will most avail him, in aiding the endeavours of youth, is that which is drawn from a strict attention to the developement of the intellectual powers and habits, and from a close and continued intercourse with his pupils, in their efforts, in their success, and in their failure. A teacher, no doubt, when he enters upon his office, must gain experience at the cost of his students, on the same principle that a young physician improves in skill, at the hazard of his patients; but in schools, where the teachers have their eyes fixed on some profession, from the moment they enter upon their duty, it is impossible

that much progress can be made by them in the difficult art of teaching. In this way there is a constant and rapid succession of inexperienced tutors thrown into the schools; and education viewed in reference to its most important objects, never can rise above a state of infancy. Teachers relinquish their office, just when they are becoming qualified to fill it. The appointment, indeed, according to the notions prevalent, is at no time considered of high estimation, it may be filled by any one who has been selected, and it is abandoned by all, whenever an opportunity offers. In such circumstances, then, we may safely infer that there can be nothing of that ardour and enthusiasm so necessary to carry a teacher through the drudgery of his professional duties. There can be no such thing as an act in education, when there are so many changes, the old and the experienced quit the helm, and the vessel is left to the direction of those who have scarcely made one voyage. In every other art, it would be thought singular indeed, if those who were appointed to teach it, were persons who, from their age or practice, had the fewest opportunities, and the most confined experience, who were to continue in that office only a very short time, who consider it merely as a temporary employment, and who moreover, during that short time, so far from having a sufficient inducement to exert their talents to the utmost of their power, would have their minds fixed on a better situation soon to be enjoyed by them, not as the reward of services in teaching, but as their reward of his attention and improvement in their favourite profession. If this would be thought absurd in every other department of life, why is an exception to be made in the case of one of the most difficult, and certainly, not the least important, of all arts—the art of teaching.

It is to no purpose to urge, in support of the present mode of employing teachers, which we wish to see reformed, that many of them have distinguished themselves by great ability and success in the discharge of their office. It would be wonderful indeed, if, among such a number as exercise that duty, and amid such a variety of genius and taste as must necessarily adorn it, there should not be found some individuals possessed of proper qualifications; who, are seen to take pleasure in communicating knowledge to youth, and in being instrumental in their progress; who do not allow their minds to be alienated from their office, by future prospects; and who find, in the consciousness of discharging a weighty obligation, a motive sufficient to support the exhausting labours with which it is attended. Such instances, however, are not to be attributed to the spirit of the system. They are rather to be viewed in the light of exceptions, and as exhibiting in strong colours, the manifold advantages which would result from a mode of appointment, calculated to secure all the talent and zeal of the teacher for the furtherance of education.

It continues to be a reproach on some individuals, that a prejudice in favour of certain modes of teaching is apt to become so powerful, as to withstand every effort to improve them; and that, while every other order of professional men is disposed not only to borrow but to steal improvements from one another. Teachers avoid all communication and intercourse, think it beneath them to take a hint which might prove useful, or to profit by the experience of those who may have ventured out of the common track. Such, is neither wise nor liberal. Engaged in the same dignified and important work, upon which the great interests of society so much depend, it ought to be the common duty of every public teacher to exert himself to the utmost, whether by adopting the *new methods*, or by improving upon the old, to raise higher and higher the intellectual and moral character of the human being. But, we forbear from insisting upon matters so obvious and so commonplace. No man doubts that it is incumbent on him to do his duty in the best way that it can be performed; the only difference of opinion is respecting the means; and, to come to a right judgment on this head, nothing more seems necessary than candid inquiry and a fair comparison. In this, as in all other questions as to right and wrong, better and worse, the power of truth is great, and must ultimately prevail.

CONCLUSION OF PHILOSOPHIC EDUCATION.

In taking a review of our strictures, we are anxious to discover whether we have sufficiently expressed what was meant to be stated relative to the beneficial effects of a system of philosophic education, calculated to improve the reason of youth and to give them a turn for reflection and inquiry. The advantage of such discipline, it is abundantly evident, must have the effect of strengthening and invigorating the faculty of reason, and of preparing it, not only as an instrument of science and literature, but for its higher office of forming rules of duty and of conduct. The daily necessities which this mode of instruction imposes upon students, of acquiring distinct notions, of attending to the grounds and evidence of their judgments, of arranging their thoughts in order, and of pursuing by analysis and induction, the links which constitute a chain of reasoning, and of expressing their ideas with perspicuity and propriety, must have the effect of producing habits of thought, deliberation, foresight and decision, in the mind. It is such a plan, or one similar to it, which philosophers, politicians, and all men of reflection, have had in view, when they have maintained that a good education is the most solid foundation of the happiness and prosperity of individuals and of nations, but, to effect this

great end, education must comprehend something more than classical literature, the knowledge of ancient or modern systems of philosophy, or of the properties of the relations of figures. It must touch the springs of feeling and of action, and contribute to the formation of intellectual and moral habits. Did all the seminaries of a nation send forth annually their hundreds and thousands, thus prepared for usefulness, who, by their example, their instruction and their influence, might produce great effects upon the opinions and conduct of others, the effect upon the general mass of human society would, we are persuaded, become far from imperceptible.

When it is farther considered that a great share of the misfortunes, disappointments, and miseries of human life, have their origin in *ignorance* or *indolence*, in *rashness* or *indecision*, in *incapacity* or *feebleness of exertion*; such a plan of education, improved as it might be, by experience and talents, must greatly tend to fortify the minds of youth against the evils with which they might be assailed in their journey through life.

Another object which has been present to our minds during the execution of these outlines, and has, we hope, been in some measure realized, was to lay down a more definite notion of what is meant by what is generally called a philosophic education. We should wish above all things, then, to impress upon the teachers of youth to keep before their eyes, a definite object, to be realized in due time by their own exertions, co-operating with those of their pupils; and in this way, affording a standard for estimating the progress which may be made, and the work which still remains to be achieved. For in such circumstances, it will be impossible to refrain from occasionally asking themselves, whether the labour they perform, and that which they require from their students, are at all likely to accomplish the end which they have in view, namely, instilling into their minds the principles of science and moral truths, based upon our HOLY RELIGION.

It appears also, that whatever improvements are made on practical education must begin with the *teachers* themselves; the art of education, like other arts, is built upon experience and observation. Improvements are not to be expected from legislators, or politicians, who have many other objects which press upon their attention; nay even from philosophers, unless they have drawn their knowledge from the above source.

It is the indispensible duty, therefore, of every one engaged in teaching, to collect *facts*, to record observations, to watch the progress of the unfolding of the human faculties, to begin the work of reformation with trials or experiments; and thus to unite their efforts in contributing to the general improvement of human reason.

Lastly, it is comfortable to think that all the improvements of which philosophic education is susceptible, may be brought about without imposing any new burden on society, and merely by adding a little to the labour, and introducing a few modifications into the system of those to whom the department of education is now entrusted.

ARITHMETICAL AND MATHEMATICAL DEPARTMENT.

(*Continued from page 349.*)

ALTHOUGH the four fundamental operations of arithmetic, may in every case be performed upon fractions, by means of the preceding rules, yet, it can hardly have escaped notice, that if the different subdivisions of unit which are employed to measure quantities less than the unit, were subjected to the same law of decrease, the calculation of fractions would have become much more convenient from the case with which one of these subdivisions might be changed into another. By making use of this law which is formed upon the basis of our system of numeration, the science of calculation has been brought to the highest degree of simplicity which it is capable of attaining.

It has been previously shown that each of the collections of units contained in any number, is composed of the units of the preceding rank, in the same manner as the ten is formed of simple units; now there is nothing to prevent this simple unit from being considered as containing ten parts, each of which will then be *a tenth*; the tenth, as containing ten parts, each of which will then be *a hundredth*; the hundredth as containing ten parts, each of which will then be *a thousandth* of unit, and so on in succession. Thus quantities as small as may be required, can be formed, by means of which, any quantities however small, can consequently be measured. These fractions, which are called decimals, because they are composed of parts of unit, each ten times less than its preceding part, may be changed into each other, in the same manner as *tens*, *hundreds*, *thousands*, *&c.* are changed into units. For,

The unit being equal to 10 tenths,
The tenth to 10 hundredths, and
The hundredth to 10 thousandths,

it follows that the tenth is equal to 10 times 10 thousandths or 100 thousandths.

For example, 2 tenths, 3 hundredths, and 4 thousandths, will be equal to 234 thousandths, as 2 hundreds, 3 tens, and 4 units, make 234 units; and it will be the same with every other case, since the subordination of the parts of unit is similar to that of the different ranks of units.

In pursuauance of this principle, decimal fractions may be written in figures, in the same manner as whole numbers, since, according to the

rule which makes a figure ten times less by being placed on the right of another, the *tenths* find their places naturally on the right of the units, the *hundredths* on the right of the tenths, and so on; but to prevent confounding the figures that express decimal parts with those which represent whole units, a point or comma, is placed on the right of the units. To express, for instance, 34 units and 27 hundredths, we write 34.27. If there were no units, their place would be supplied by a cypher, and so would the places of all the decimal parts, which might be wanting to complete the succession of those expressed in the number proposed.

Thus 19 hundredths are written 0.19
304 thousandths 0.304
3 thousandths 00.03

On comparing the expressions of the decimal fractions above with the following, 19-100, 304-1000, 3-1000, which result from the usual method of representing a fraction, it will be seen that, *to represent in its whole form, a decimal fraction written as a vulgar fraction, the numerator of the vulgar fraction must be taken as it is, and placed in such a way that, as many figures as there are cyphers after the unit in the denominator, may come after the decimal point.*

Reciprocally *to change a decimal fraction written in the whole form, into that of a vulgar fraction, it must receive, as a denominator for the figures of which it consists, an unit followed by as many cyphers as there are figures on the right of the decimal point.*

Thus, the fractions 0.56 and 0.036 become 56-100 and 36-1000.

Numbers containing decimal parts are expressed in words, by reading first, the figures placed on the left of the decimal point; next, those on the right of it, taking care to add to the latter, the denomination of the parts which they represent.

The number 26.736 is read 26 units 736 thousandths. The number 0.0673 is read 673 ten thousandths, and 0.0000673 is read 673 ten millionths.

As decimal figures receive their value only from the rank which they hold in relation to the point, any number of cyphers may be either written or erased at the right, indifferently. For instance, 0.5 is the same as 0.50; and 0.784 the same as 0.78400; for in the first case, although the number which expresses the decimal fraction has become ten times greater, yet, the parts themselves have become hundredths, and consequently ten times less than at first; in the second case, the number which expresses the fraction has become an hundtimes greater, but the parts having become hundredthousandths, are a hundred times less than at first; this transformation then returns to the same as that which is wrought upon a vulgar fraction when both its terms are multiplied at once by the same number, and the suppression of the cyphers, would be the same as dividing them both by the same number.

The addition of decimal fractions and of the numbers with which they may be connected, requires no other rule than that of whole numbers, since the decimal parts are composed of each other, in going from the left to the right, in the same manner as the different ranks of whole units.

For instance, let the numbers to be added, be 0.56, 0.003, 0.958; disposing them as follows:

0.56
0.003
0.958
Sum 1.521

Their sum is found to be 1.521.

Again, let the numbers be 19.35, 0.3, 84.5 and 110.02, which contain whole units, they will be arranged thus:

19.35
0.3
84.5
110.02
Sum 214.17

And their sum will be found by the same means to be 214.17.

In general, *the addition of decimal numbers is performed like that of whole numbers; observing to place the decimal point in the same column of the sum, as it holds in each of the numbers given to be added.*

The rules given for the subtraction of whole numbers, likewise answer, as will be shown, for the subtraction of decimals For example, let it be required to subtract 0.3697 from 0.62; it will be observed, that the last number which only contains hundredths, while the first contains ten-thousandths, may likewise be converted into ten-thousandths by placing two cyphers on its right, making it 0.6200. The operation will then be arranged as follows:

0.6200
0.3697
difference 0.2503

And the difference is found to be 0.2503.

Again, let the number 7.364 be subtracted from 9.1457; the operation being arranged thus:

9.1457
7.3640
difference 1.7817

The difference marked below is found. The cypher on the right of the number to be subtracted, might have been dispensed with, provided that its different figures had been placed under those of the upper number, expressing corresponding parts, or orders of units.

In general, *the subtraction of decimals is performed like that of whole numbers, remembering to equalize the decimal places in the two numbers proposed, by annexing on the right of that which has the least, as many cyphers as are required to complete it; and to place the decimal point of the difference in the same column which it occupies in the given numbers.*

The proofs of addition and subtraction of deci-

mal numbers are performed precisely like those of the same operations upon whole numbers.

Since the decimal point separates the collections of whole units from the decimal parts, to change its place will be to change the value of the entire expression. By moving it towards the right, the figures which were before in the fractionary part, pass into the whole number, and, consequently increase the aggregate value of the number proposed. On the contrary, by moving the decimal point towards the left, the figures of the part expressing the whole units, pass into the fractionary part, and, consequently diminish the value of the expression proposed.

The first change renders the given number ten, an hundred, a thousand, ten times greater, accordingly as the decimal point is moved one, two, three, &c. places towards the right; since that, every time the decimal point is thus moved one step, all the figures, relatively to this point, advance one step towards the left, and thereby receive a value ten times greater than they possessed at first.

If, for instance, in the number 134.28, the decimal point be transferred to the place between the 2 and the 8, we have 1342.8; the hundreds will have become thousands, the tens hundreds, the units tens, the tenths units, and the hundredths tens. All the parts of the number having become ten times greater, it will be the same as if the whole had been multiplied by ten.

The second change renders the given number ten, a hundred, a thousand &c. times less, accordingly as the decimal point is removed one, two, three, &c. places towards the left; because, at every time that the point is thus removed one place, all the figures relatively to this point, are moved one step towards the right, and thereby receive a value ten times less than they possesed at first.

If in the number 134.28, the decimal point be placed between the 3 and the 4, we shall have 13.428; the hundreds will have become tens, the tens units, the units tenths, the tenths hundredths. All the parts of the number having thus become ten times less, it is the same as if the tenth part of the whole had been taken, or it had been divided by ten.

From the foregoing considerations, the advantage which decimal fractions possess over ordinary fractions, is immediately apparent: since all the multiplications and divisions which in vulgar fractions must be wrought upon the denominator, may be effected in decimals, either by the addition or suppression of a certain number of cyphers, or by simply changing the place of the point. The theory of decimal fractions as well as the manner of multiplying and dividing them, may be generally deduced from the theory of ordinary fractions, by adapting to them these modifications: but, the following considerations lead more directly to this end.

Let us first suppose, that there be decimal places in the multiplicand alone. If the point be taken away, it will become ten, an hundred, a thousand, &c. times greater, according to the number of its decimal places; and the product resulting in this case, from multiplication will be a like number of times greater than the product sought; this latter product may then be obtained by dividing the other by ten, a hundred, a thousand, &c. which is effected by separating from the right as many decimal places, as there were in the multiplicand.

If, for example, it were required to multiply 34.137 by 9, the product of 34.137 by 9, would be first sought, which would give 307.233; and, as the suppression of the point would have made the multiplicand a thousand times greater than it was before such suppression, the product found should be divided by a thousand, otherwise the three last figures should be separated from its right; in this manner, we should obtain 307.233.

In general, *to multiply any number containing decimal places by a whole number, remove the decimal point from the multiplicand; but, separate from the right of the product as many decimal places as there were in the multiplicand.*

Whenever the multiplier contains decimal places, and the decimal point is removed, it is made ten, an hundred, a thousand, &c. times greater, according to the number of its decimal places; if it be employed in this condition, it will evidently give a product, ten, a hundred, a thousand, &c, times greater than the product sought, which may consequently be obtained by dividing the first product by one of these numbers; that is to say, by separating from the right of the product, as many decimal places as the multiplier contains, or if the multiplicand also contain decimal places, by advancing the point a like number of places towards the left.

Let it be required, for example, to multiply 172.84 by 36.003; on removing the point from the multiplier alone, the product would be, (according to the preceding remarks,) 6222758.52; but the multiplier being a thousand times greater than it should have been, this product must be divided by a thousand, or, the point must be removed three places towards the left; this gives 6222.75852 for the required product, in which there are consequently as many decimal places as there are both in the multiplier and the multiplicand. In general, *to multiply two numbers containing decimal places together, the point should be removed from each; but as many decimal places as both contain, must be separated from the right of the product.*

In some cases, one or more cyphers must be placed on the left of the product, in order to complete the number of the decimal places which it ought to contain, according to the foregoing rule. If, for example, 0.624 were required to be multiplied by 0.003, on forming the product of 624 by 3, the result would be the number 1872, which consists only

of four figures, and as the rule requires the separation of 6 decimal places, this cannot be performed, but by placing three cyphers on the left of the number, one of which will hold the place of the unit, and the exact product will be 0.001872.

It is plain, that the quotient of two numbers does not depend at all upon the absolute magnitude of their units, provided it be the same in both. If it were required to divide 451.49 by 13, it would be observed, that the first number is equal to 45149 hundreths, and the second to 1300 hundreths, and that these fractional parts must give the same quotient, as if they were entire units. We should thus be led to remove the point in the first of the proposed numbers, and to place two cyphers on the right of the second; and nothing would remain but to divide the number 45149 by 1300, which would give 34 (of a whole number) 949-1300 (of a fraction) for the quotient.

The conclusion from this is, that *to divide a number containing decimal places by a whole number, the point in the dividend must be removed, and as many cyphers as there are decimal places in the dividend, must be placed on the right of the divisor; no change then need be made in the quotient.*

Whenever the dividend and the divisor both contain decimal places, previously to the removal of the point, there should be placed on the right of that one of the two numbers which contains the fewer decimal places, as many cyphers as are necessary to make it terminate in the same rank of decimals as the other. Since the two numbers proposed will then express the same parts of the unit, they will give the same quotient, as if they expressed entire units. For example, let it be required to divide 315.432 by 23.4; the latter number must be changed into 3.400 and then it will remain to divide 315.432 by 23.400; the quotient will be 13 (of a whole number) 11232-23400 (of a fraction.)

Thus *to divide by each other two numbers containing decimal places; as many cyphers as will make the number of decimal places equal in the divisor and dividend, must be placed on the right of whichsoever contains the fewer decimal places; then, after the removal of the point, there will be nothing to change in the quotient.* Since the only object of recurring to decimal fractions, is to avoid the use of ordinary fractions, it is natural to employ them for the purpose of approximating to quotients which cannot be exactly obtained; this is accomplished by changing the remainder into tenths, hundreths, thousandths, &c, and any other decimal parts that will contain the divisor; as in the following example:

```
1300) 45149 (34.73
      3900
      ----
       6149
       5200
       ----
        949 remainder of Units
        9490 reduced to Tenths.
        9100
        ----
         390 remainder of Tenths.
         3900 reduced to Hundreths.
         3900
         ----
```

After having reached the remainder 949, a cypher is added on the right to multiply it by ten, or to change it into tenths; there is thus formed a new partial dividend, composed of 9490 tenths, and giving for a quotient 7 tenths, which is written on the right of the decimal point, where the units terminate. There remain still 390 tenths, which are reduced into hundredths, by the addition of a new cypher, forming a second dividend composed of 3900 hundredths, and giving for the quotient 3 hundredths, which are placed on the right of the tenths. Here the operation terminates, giving 34.73 for the exact result. It might have been carried farther, had a third remainder been left, by changing this remainder into thousandths, and prosecuting the division and the reduction in the same manner, until there should have been obtained either an exact quotient, or, a remainder consisting of parts so small as to be considered of no importance.

It should be remarked, that the point must be placed after the entire units of the quotient, as in the preceding example, to distinguish them from the decimal places, the number of which will be equal to the number of cyphers successively written on the right of the remainders.

The numerator of a vulgar fraction, after having been changed into decimal parts, may be divided by the denominator, as in the above example, and thus the fraction will be changed into decimals. Let the fraction be, for instance, $\frac{1}{8}$; the operation will be performed in the following manner;

```
   1
8) 10 (0.125
   8
   --
   20
   16
   --
    40
    40
    --
```

Suppose, again, that the fractions given were 4-797; the numerator must be changed into thousandths before the division can be commenced:

```
     4
797) 4000 (0,005018
     3985
     ----
       1500
        797
       ----
        7030
        6376
        ----
         654
```

To whatever extent this last division may be carried, an exact quotient can never be obtained, because the fraction 4-797 cannot be rigorously expressed in decimal parts, like the fraction $\frac{1}{8}$.

This difference results from the fact, that the denominator of any fraction, which does not divide its numerator, cannot give an exact quotient, except when it divides one of the numbers 10.100, 1000, &c, by which the numerator is successively multiplied; since, according to a principle demonstrated in Algebra, no number can divide any product, unless the factors of the number divide those

of the product. Then, the numbers 10, 100, 1000, &c. being all formed from the number 10, the factors of which are 2, and 5, are only divisible by numbers formed from the same factors; 8, comes within this case, since it results from 2, by 2, by 2. Vulgar fractions which cannot be rigorously valued in decimals, exhibit in their approximate expression, when carried out to a sufficient extent, a character which enables us to return from the decimal to the vulgar form; we refer to the periodical return of the same figures. The fraction 12-37, on being changed into decimals is found to be equal to 0.324324, and the figures 3, 2, and 4, will always return in the same order, without ever bringing the operation to a conclusion.

In fact, as there can never be in any division, any other remainder than some one of the whole numbers which are less than the divisor, it follows, necessarily, when more divisions have been performed than there are of these numbers, that some one of the former remainders must recur, and that, consequently, the partial dividends must return in the same order. In the above example, three divisions are sufficient to prevent this recurrence; but six are required for the fraction 1-7, because then the six numbers below 7, have been found for remainders and there results 0.1428571. The fraction $\frac{1}{3}$, leads only to 0.3333. Fractions which have any number whatever of 9's, have but one significant figure, viz. 1 in their period;

$\frac{1}{9}$ gives 0.1111

$\frac{1}{99}$ 0.0101

$\frac{1}{999}$ 0.001001001

and so on with others, since each partial division being required to be performed upon the numbers 10, 100, 1000, &c. must always leave unit for the remainder.

Profiting by this remark, we can pass without difficulty, from a periodical decimal fraction, to the vulgar fraction from which it is derived. It is evident, for example, that 0.3333 returns to the fraction 0.1111 multiplied by 3; and since this last is but the developement of 1-9, it may thence be concluded that the first is the same as 1-9 multiplied by 3, or as 3-9, or finally as 1-3. Whenever the question respects fractions of which the period is composed of two figures, they must be compared to the developement of 1-99, when of three to that of 1-999, and so on in succession.

Take 0.324324 for an example; it is plain, that this fraction would result from multiplying 0 001001 by the number 324; then, on multiplying 1-999 of which this last is the developement by 324, we obtain 324-999 for the first; and the two terms of this result being divided by 27, the fraction 12-37 recurs.

In general, *the vulgar fraction from which any periodical or circulating decimal is formed, may be found by placing as a denominator, under the number which expresses a period, as many nines as there are figures in the period.*

Where the period of the fraction does not commence at the first decimal figure, the point may be for a moment placed immediately before the first figure of the period, and may then be valued, beginning from this figure, and considering those on the left of it as units; nothing then will remain but to divide the result by 10, 100, 1000, &c. according to the number of places that the point may have been removed towards the right. The fraction 0.324141 for example, is first to be written thus 32.4141; the part 0.4141 answering to 41-99, there results 32 of a whole number and 41-99 of a fraction, which must be divided by 100, since the point was removed two places towards the right, and from which consequently comes 32-100 and 41-9900; after having reduced these fractions to the same denominator, and having added them we arrive at 3209-9900, a fraction which will reproduce the given developement. In order to form a precise notion of the nature of these expressions, it is sufficient to consider the fraction 0.999. On an attempt to recur to its primitve value, it is found to answer to 9 divided by 9, which is nothing else than unit; nevertheless, whatever be the number of figures to which the expression may be extended, the unit can never be formed from it; if we limit ourselves, to the first, it will fall short by 1-10, if to the second by 1-100, if to the third by 1-1000, and so on; thus we may be perpetually approaching the unit, without ever being able to reach it. The unit then is here but a limit, to which the expression 0.999 approaches in proportion to the number of figures of which it consists.

These are all the essential rules of the arithmetic of abstract numbers; they embrace whatever relates to the valuation, the addition, the subtraction, the multiplication, and the division of units, or precise or approximate parts of units of whatever description, provided that the units presented for any given calculation, be considered as of the same description in that particular calculation.

OF PROPORTIONS.

The different methods of performing the four fundamental arithmetical operations of addition, subtraction, multiplication and division upon all numbers whether whole or fractional, have been exhibited in the preceding part of this treatise; and all questions relative to numbers, ought to be considered as resolved, when after an attentive examination of the terms of the questions, they become reduced to some one of these forms. Whatever, belongs to arithmetic, might consequently be terminated here; for the rest is, properly speaking, peculiar to Algebra: it may be better, however, to solve a few questions, which, by exercising

students upon what has already been shown, will prepare them for Algebraical analysis, and conduct them to the important theory of ratios and proportions, which is usually included in arithmetic.

A piece of cloth containing 13 yards was bought for 130 dollars; what would be the cost of a piece of the same cloth which should be 18 yards in length?

It is plain, that if the price of a single yard of the cloth were known, on repeating this price eighteen times, the result would be the price of the piece consisting of 18 yards; now, since 13 yards cost 130 dollars, a single yard would cost a thirteenth part of 130 dollars, or 130-13: the result 10 dollars is obtained, after having performed the division, and on multiplying this number by 18, there comes 180 dollars for the sum required. This is, in fact, the price of the piece of eighteen yards.

A person travelling constantly at an equal pace, goes 5 miles in three hours; the question is, how many miles would he go in 11 hours?

By reasoning as in the former example, it will be seen that this traveller will pass over, in a single hour $\frac{1}{3}$ of 5 miles, or 5-3, and that, in 11 hours, he would go 11 times as far, or 5-3ds of a mile multiplied by 11, equal to 55-33, equal to 18 miles and $\frac{1}{3}$? *Suppose again, that it were required to ascertain in how long a time the traveller, as in the preceding question, would go 22 miles?*

It will be perceived, that if the time that the traveller would occupy in going a single mile, were known, the number of hours sought would result from multiplying this time by 22; now the traveller in question, occupying 3 hours in travelling 5 miles, will only employ 1-5 of this time or 3-5 of an hour, in travelling 1 mile; this number being multiplied by 22, gives 66-5, or 13 hours and 1-5; and since an hour consists of 60 minutes, we have 13 hours and 12 minutes in the place of 13 hours and 1-5. The unknown quantity in each of the preceding questions has been discovered by the analysis of the question itself; but in all these questions, the known numbers and the numbers sought, depend upon each other in a manner which it is worth our while to examine.

For this purpose, I resume the first question, the object of which is to ascertain the price of 18 yards of cloth, when 13 yards cost 130 dollars. It is clear, that the sum to be paid for a piece of this cloth, would become double, if the piece contained a number of yards double of the first piece; that if this number became triple, the price would be likewise trebled, and so on; besides, that for one half, or two thirds of the piece, but one half or two thirds of the whole price would be paid.

According to these notions, which all who understand the nature of the terms will admit without hesitation, it will appear that if there be two pieces of the same cloth, the second will contain the first, as often as the price of the second will contain that of the first; and this circumstance is expressed by saying, that the prices are in *proportion* to the lengths, or are to each other in the same *ratio* as the lengths. This example will serve to fix the sense of several expressions of frequent recurrence.

The ratio of any two lengths is the number, whether whole or fractional, which shows how often one of the lengths contains the other. If the first piece contained 4 yards, and the second 8, the ratio of the second to the first would be 2, since 8 contains four 2 times. In the first example, the first piece contained 13 yards, and the second 18; the ratio of the second to the other then was 18-13, or 1 of a whole number, and 5-13 of a fraction. *In general, the ratio of the two numbers, is the quotient of one by the other.*

As the prices are in the same ratio to each other as the lengths, it follows, that 180, the price of the second piece, being divided by 130, the price of the first, must give 18-13 for the quotient, which is, in fact, the case; for, on reducing 180-130 to its simplest expressions, we have 18-13.

The four numbers 13, 18, 130, 180, then written in this order are such, that *the second contains the first, as often as the fourth contains the third; and they thus form what is called a proportion.*

The numbers 13, 18, 130, and 180, are called the terms of the proportion. It may be also said, that *a proportion is formed by uniting two equal ratios.*

It must be here observed that a ratio is not changed by multiplying or dividing its two terms by the same number; and this is evident, because a ratio being always the quotient of a division, may always be put in a fractional form. Thus the ratio 18-13 is the same as 180-130.

The same considerations apply likewise to the second example. The traveller that goes 5 miles in 3 hours, will go twice the distance in twice the time, three times the distance in three times the time; so, 11 hours, the number that expresses the time which this traveller has employed in going 18 miles and 1-3, or 55-3 of a mile, should contain 3 hours, the time that he takes in going 5 miles, as often as 55-3 contain 5; the four numbers 5, 55-3, 3, 11, are then in proportion; and in fact, if 55-3 be divided by 5, the result will be 55-15, which is equal to 11-3. Any case in which a proportion exists among four numbers, may from this be easily recognized. In order to show that the four numbers 13, 18, 130, and 180, are in proportion, they are written thus; 13 : 18 : 130 : 180; and are read thus: 13 is to 18 as 130 is to 180, which means, that 13 is the same part of 18 as 130 is of 180, or that 13 is contained in 18 as many times as 130 is contained in 180, or lastly, that the ratio of 18 to 13 is the same as that of 180 to 130.

The first term of the ratio is called the *antecedent*, and the second the *consequent*. In a proportion, there are two antecedents and two consequents, namely; the antecedent of the first ratio and that

of the second as well as the consequent of the first ratio and that of the second. In the proportion 13 : 18 : : 130 : 180, the antecedents are 13, 130 ; the consequents 18, 180. The consequent of the ratio will hence forward be taken for the numerator of the fraction which expresses the ratio, and the antecedent for its denominator.

To ascertain that there exists a proportion between the four numbers 13, 18, 130, and 180, the fractions 18-13 and 180-130 must be examined to see whether they are equal, and for that puspose, the second must be reduced to its simplest expression ; but the same verification may be made by observing that, that if the two fractions 18-13 and 180-130 be equal as they are supposed to be,it follows that, on reducing them to the same denomination, the numerators will become equal, and consequently that 18 multiplied by 130 will give the same product as 180 by 13. This in fact, is the case ; and the reasoning which shows it, being independent of the particular value of the numbers, proves *that if four numbers be in proportion, the product of the first and last, or the two extremes, is equal to that of the second and third, or of the two means.*

It may, at the same time, be gathered, that if the four given numbers were not in proportion, they could not have the property above expressed ; for, the fraction, that expresses the first ratio, not being equal to that which expresses the second, the numerator of the one, will not be made equal to that of the other, on reducing them both to the same denominator. The first consequence naturally deducible from this, is, that the order of the terms of a proportion may be changed, provided that the proportion substituted, be such, that the product of the extremes remain equal to that of the means. In the proportion 13 : 18 : 130 : 180 ; then the following arrangements may be made :

13:	18: :	130:	180
13:	130: :	18:	180
180:	130: :	18:	13
180:	18: :	130:	13
18:	13: :	180:	130
18:	180: :	13:	130
130:	13: :	180:	18
130:	180: :	13:	18;

for each of them the product of the extremes and the means continue to be formed of the same factors. The second arrangement, in which the means change places with each other, is one of those the most frequently practised.

This shows, that the two antecedents or the two consequents of any proportion may be divided by the same number, without disturbing it, for this change makes the first ratio from the two antecedents, and the second from the two consequents. If we had, for instance, 55 : 21 : : 165 : 63, by changing the places of the means, there would come 55 : 165: 21 : : 63; the two terms which form the first ratio, may then be divided by the number 5, which gives 11 : 33 : : 21 : 63 : on changing the places of the means again, we get 11 : 21 : : 33 : 63, a proportion true in itself, and differing only from the proportion given in this, that its two antecedents have been divided by 5.

Since the product of the extremes is equal to that of the means, one may be taken instead of the other ; and as by dividing the product of the extremes by one of them, we should obtain the other for the quotient, it follows that, *by dividing the product of the means by one extreme, the other extreme is also found.* And on the same principle, *by dividing the product of the extremes by one of the means, the other mean is also found.*

Any term of the proportion may thus be found, when three others are known ; for the term sought must be either one of the extremes, or one of the means. The first question given may be resolved by one of the above rules. In fact, as soon as it is understood, that the prices of the two pieces of cloth are in proportion to the quantities contained in each, the proportion is thus written :

13 : 18 : · 130 : x

putting the letter x to hold the place of the required price of the piece of 18 yards : and this price which is one of the extremes, is found by multiplying the two means 18 and 130 together, which gives 2340, and dividing this product by the known extreme 13 ; the result obtained is 180.

The operation by which the fourth term of a proportion is found when the three first are given, is called the *rule of three.* The authors of most of the books of arithmetic, have distinguished it into several kinds ; but this distinction is entirely useless to those who comprehend thoroughly in what proportion consists,and understand well the expression of any given question. Some applications will explain this.

A labourer having accomplished 217.5 feet of work in 9 days it is required to tell how much time he would take to perform 423.9, supposing that he works always at the same rate ? In this question, the unknown term is a number of days that will contain the nine days, occupied in performing 217.5 feet as many times as 423.9 contains 217.5 ; we have the following proportion ;

217.5 : 423.9 : : 9 : x, and x is found to be 17.54 feet.

The only difficulty that can occur in any question whatever, consists in the manner of establishing the proportion ; certain rules for forming it in every case are here subjoined. Among the four terms which compose any proportion, there are always two numbers which are of the same kind, and two other numbers which are also of the same kind, but different from the first kind. In the first example, two of the terms expressed, are yards, and the other two are days.

The two terms of each kind must then first be distinguished ; and when that shall have been done,

we have necessarily the quotient of the greater term of the second kind, divided by the less term of the same kind, equal to the greater term of the first kind, divided by the less term of the same kind, which will give this proportion.

The less term of the first kind,
is
To the greater term of the same kind,
as
The less term of the second kind,
is
To the greater term of the same kind.

In the foregoing example, this rule gives at once, 217.5 : 423 : : 9 : x ; for the unknown term must be greater than 9, because, the more work that there is to be done, the greater the number of days that must be taken to do it in.

If it were required to find how many days 27 labourers would take to execute a work which 15 labourers, working at the same rate as the former, have performed in 18 days, it would be seen that the greater the number of labourers, the less must be the number of days, and reciprocally. There is still a proportion in this case, but in the inverse order : for if the number of labourers of the second set were triple that of the first set, for example, the labourers of the second set, would require but a third part of the time employed by the first set ; the first number of days then must contain the second, as often as the second number of labourers contain the first.

The order in which these quantities contain each other, being the inverse of that which is assigned them by the expression of the question, it is said, that the two numbers of labourers are in *inverse ratio* to the two numbers of days. If the two first and the two last be compared in the order in which they present themselves, the ratio of the first will be 3, or 3-1 and that of the others will be $\frac{1}{3}$, a fraction which is the inverse of 3-1. It should be observed, that a ratio is inverted by inverting the fraction that expresses it ; since the antecedent is thus made to take the place of the consequent, and the consequent to take that of the antecedent—3-2 or 2 : 3 is the inverse of 2-3 or 3 : 2.

The rule last given, simplifies these considerations materially ; for by taking the two numbers of workmen for the quantities of the first kind, and the two numbers of days for those of the second, and placing those of each kind according to the order of their magnitude, we have the following proportion 15 : 27 : : x : 18, from which we obtain x, equal to ten.

FINIS.

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